https://doi.org/10.17221/109/2023-JFS

Calculation and operational assessment of tyre contact areas in the tractor-and-trailer unit

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Citation: Marušiak M., Zemánek T., Neruda J., Nevrkla P. (2024): Calculation and operational assessment of tyre contact areas in the tractor-and-trailer unit. J. For. Sci., 70: 144–159.

Abstract: This paper deals with the verification of models for the calculation of the contact area with the soil using data measured during the testing of a tractor-and-trailer unit. The main emphasis was put on the method of calculating the contact area by means of a super ellipse. The comparison included calculation models with the input parameters of contact length and contact width of the tyre. These parameters were compared with values measured in the field where the main variables were tyre dimensions, inflation pressure and load. Results of comparisons show that the method of calculating the contact area using a super ellipse exhibited the best match with the measured values of all compared calculation models (81% in super ellipse with measured half-axes and 75% in super ellipse with calculated half-axes). As to trailer tyres, the match of measured values and those calculated using a super ellipse was even 95%. In the second step, also some empirical models for calculating the contact area were compared with the measured data, not entered by contact length and contact width as variables. Some of these models show a very good match with the measured data, which can be compared, or it is even higher than the calculation by means of a super ellipse. With the specified tyre deformation, however, we consider the model of calculation using a super ellipse as more appropriate for determining the size of contact area as it focuses on the geometry of tyre contact with the ground.

Keywords: calculation model; forest machinery; super ellipse; tyre footprint

Travel assembly of forest and agricultural machines comes into contact with the soil surface of the forest stand on the bearing surface (contact area), the size and shape of which depend on multiple factors, e.g. tyre dimensions and type, inflation pressure, wheel load and also on soil characteristics. The contact area is important in terms of the efficiency of engine performance transfer on the soil surface as it affects the magnitude of shearing stress in the soil, induced by the transfer of the driving torque. Contact pressure between the tractor and the ground, which affects undesirable soil compaction, is decreasing as the contact area increases. This is important particularly when driving across loose soil or on soil with higher moisture content, which is the most susceptible to compaction (Šmerda, Bauer 2010).

The issue of the determination of tyre contact area size has been studied by multiple authors who developed calculation models based on their

Supported by the Technology Agency of the Czech Republic, Grant No. TH02010115.

https://doi.org/10.17221/109/2023-JFS

observations and measurements. The most commonly used of them for practical calculations is the vertical projection of the tyre footprint in the soil of poor carrying capacity or the tyre contact area on solid ground.

When measuring the footprint of a tyre with the protruding pattern blocks, it is necessary to distinguish between ideal (projection of tyre footprint), actual and effective contact areas (Figure 1). In general, the area of the tyre footprint is also considered the tyre area between the pattern blocks in spite of the fact that it is not completely in contact with the base of the hard surface. The measured values of effective footprint areas are used in more accurate calculations and in determining the shear stress in the soil during the transfer of wheel drive torque. The size of the ideal footprint area is overrated compared with the real area and is used in some calculation models based on the average load of wheels (Saarilahti 2002).

There are multiple empirical models dealing with the calculation of the contact area between the tyre and the soil with low bearing capacity that show an increased contact area when the bearing capacity of the soil is decreased, e.g. Komandi (1990), Schwanghart (1991), or Grečenko (1995). When the tyre is buried in the soil with low bearing capacity, a part of the load is also transferred by the sides of the tyre, and the bearing surface is represented by the general area (Pacas 1983). Calculation models of other authors (Söhne 1958; Wulfsohn, Upadhyaya 1992) also show that at the given inflation pressure and load, the contact area increases by decreasing the soil rigidity. Diserens (2009) performed regression calculations to evaluate the contact area. The author obtained the best evaluations from multiple regression calculations from measurements identifying tyre structure (cross-ply or radial), tyre width for the cross-ply set and tyre type for the radial set (low-profile or terra profile). Variables such as tyre dimension (product of width and overall diameter), wheel load and inflation pressure were also highly significant. Anifantis et al. (2020) used an experimental, numerical approach for modelling the mechanical behaviour of a tyre for agricultural machines. The response surface methodology was applied to find two mathematical regression models, useful for studying the variations of tyre footprint dimensions according to the type of tyre.

When the tyre is in contact with solid ground, the contact area size decreases with the increasing tyre stiffness caused by increased inflation pressure (Koolen et al. 1992). Diserens et al. (2011) dealt with the mathematical determination of the contact area of agricultural traction tyres on firm soil. Based on field measurements and using tyre size, load and inflation pressure, the authors determined three classes of tyres for evaluating the contact area. Grečenko (1995) determined the tyre footprint area on the hard ground using catalogue values. The author specified a correction factor depending on the actual tyre load to assess the footprint area of a tyre with arbitrary loading and inflation pressure.

When calculating the footprint area (contact area), it is also necessary to determine the shape of its contour. Upadhyaya and Wulfsohn (1990) developed a mathematical model for the tyre contact area on the solid ground, based on wheel geometry and tyre deformation. Results of calculation relations of the model show that the contour of the footprint area of a tyre with minor deformation is an ellipse and that it approaches a rectangle

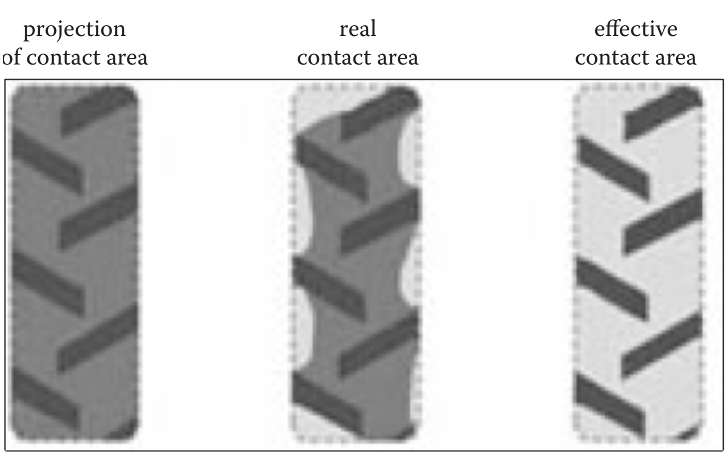

Figure 1. Tyre contact area on the soil with poor carrying capacity (Abeels 1994)

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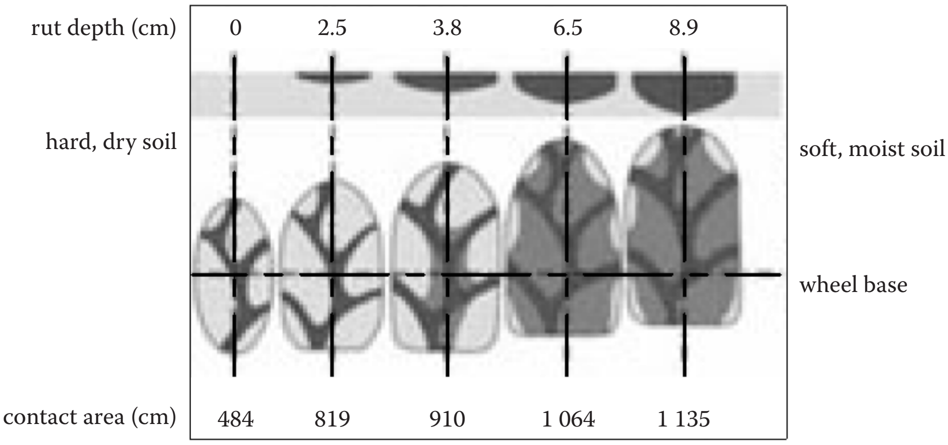

Figure 2. Tyre contact area and rut depth at different values of soil moisture content (Hallonborg 1996)

if the deformation increases. According to Pacas (1983), the shape of the contact area ranges from circular to rectangular according to the tyre type and inflation pressure. Błaszkiewicz (1990) determined the contact area between the tyre and the ground with the help of a mathematical model. The model used a rectangular coordinate system to determine the coordinates of points in the contact area.

In agricultural and forest machines, calculation models were also applied for determining the size of the tyre footprint area, based on the assumption that the shape of the contact area contour is a super ellipse (for the concept definition see the chapter Material and Methods). This method of determining the contact area is used by many authors. For example, Hallonborg (1996) considered the super ellipse to be the best way how to describe the form and shape of the contact area which may vary from a circle through an ellipse to a square or even a rectangle. Figure 2 provides an example of the tyre contact area and rut depth at different values of soil moisture content. Keller (2005) assumed that the contour of the contact area has the shape of a super ellipse where the longitudinal and transverse axes of the tyre footprint area are axes of symmetry. Schjønning et al. (2008) developed a calculation model called FRIDA dealing with the distribution of pressure in tyres that are in contact with the ground. This model makes use of a super ellipse for describing the contact area shape. Roşca et al. (2014) predicted the traction force and traction efficiency for a 2WD agricultural tractor, assuming the shape of the tyre-ground contact area to be a super ellipse.

Ptak et al. (2022) studied the deformation of bias-ply and radial tyres under variable vertical load and inflation pressure using the 3D scanning method. The 3D scanning method was also used by Farhadi et al. (2018) when investigating the effect of changes in the tractor tyre contact volume on rolling resistance. Derafshpour et al. (2019) used a system based on image processing for the real-time assessment of the dynamic tyre contact area. Melzi et al. (2014) performed a numerical analysis of the influence of tyre characteristics on the driving comfort of an agricultural vehicle using a multi-body model. The tyre contact surface was computed according to the tread pattern geometry.

This paper deals with a comparison of models calculating the tyre contact area with the ground using data obtained during the testing of a tractor-and-trailer unit. The goal of the comparison is to find a calculation method by means of which the size of the tyre contact area could be calculated based on tyre dimensions, its deformation and inflation pressure.

MATERIAL AND METHODS

Theoretical part

The comparison of tyre contact area calculations with the measured values will make use of calculation models developed by several authors. In order to be able to compare the calculation models with each other, all selected models of contact area calculation work with the same input parameters (contact length, contact width and tyre deformation). As part of the discussion, three more models will be included in the comparison which do not work with these input parameters but use e.g. statistical data obtained from several measurements. At the same time, the comparison will also include the method of the contact area calculation

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by means of a super ellipse, where contact length and contact width will be used in the first step, measured from the tyre footprint (projection of the tyre tread part when lamellas are in contact with the solid ground). In the next step, the comparison with measured values will include also the processing of contact length and contact width. The values exhibiting the best match with the measured data will be used for the calculation of the super ellipse. The calculation models, diagrams and statistical evaluation will be run in the MS Excel program (Version Microsoft Office Professional Plus, 2016). When analysing the results of the measurements, the percentage deviations of the measured data and the results of the calculation models will be first calculated for all variants of the tyre/load. Then the arithmetic mean of the differences in the measured values and deviations will be calculated for each model, which will indicate the agreement with the measured values. Calculation models to be used in the comparison are as follows:

The super ellipse model. A number of curves such as a circle, ellipse, but also a square or rectangle can be expressed by the Equation (1) introduced by the French mathematician Gabriel Lamé in 1818:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1 \tag{1}$$

where:
a, b – half-axes of the super ellipse (m);
x, y – coordinates of the super ellipse (m);
n – exponent of the super ellipse.

In the orthogonal coordinate system, the super ellipse exponent n is a positive real number which determines the super ellipse shape, while the parameters a and b are half-axes which determine its size. By changing the value of the exponent n, Lamé obtained a wide spectrum of curves. A curve for $n = 2$ is an ellipse (for $a = b$ it is a circle). If the value of n is decreasing to 1, the curve creates a peak on the top; a curve for $n = 1$ is a parallelogram (diamond). For $n > 2$, the curve sides are flattened and the curve shape starts to resemble a rectangle. The Danish mathematician P. Hein suggested that Lamé curves would be referred to as the super ellipse. Calculation of the super ellipse area requires using the so-called gamma function Γ (Spíchal 2020), see Equation (2):

$$S = 4 \times a \times b \times \frac{\left[\Gamma\left(1 + \frac{1}{n}\right)\right]^2}{\Gamma\left(1 + \frac{2}{n}\right)} \tag{2}$$

where:
S – super ellipse area (m^2).

Keller (2005) developed a relation that is used for the calculation of the super ellipse exponent n based on the measurement of the tyre contact pressure. The relation expresses the dependence of the super ellipse exponent n on tyre dimensions, see Equation (3):

$$n = 2 \times 1 \times \left(b_P \times d\right)^2 + 2 \tag{3}$$

where:
b_P – tyre width (m);
d – tyre diameter (m).

The use of the relation for the calculation of the exponent n in the equation of the super ellipse showed a good match with the measured data of the contact area (Diserens 2009; Rosca et al. 2010). Assuming that the longitudinal and transverse axes of tyre contact area are axes of the super ellipse symmetry (Keller 2005), its half-axes a and b will have half values of the contact area length and width, as described by Equation (4):

$$a = \frac{l_c}{2};\; b = \frac{b_c}{2} \tag{4}$$

where:
l_c – total length of the tyre contact area – contact length (m);
b_c – max. width of the tyre contact area – contact width (m).

Modifying Equation (1), we get the relation of the dependence of the coordinates y and x, needed for the construction of the super ellipse, as described by Equation (5):

$$y = b \times \left[1 - \left(\frac{x}{a}\right)^n\right]^{\frac{1}{n}} \tag{5}$$

Model according to Schwanghart (1991). This model is designed based on the results of testing

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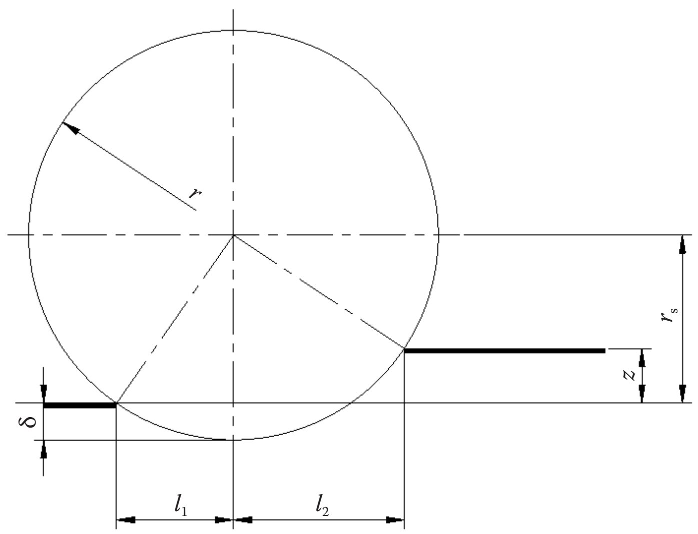

Figure 3. Elastic tyre on a soft surface (Schwanghart 1991)

δ – tyre deformation (m); l_c – contact length (m), $l_c = l_1 + l_2$; r – radius of unloaded tyre (m); r_s – radius of loaded tyre (m); z – wheel sinking [rut depth (m)]

four types of tyres sized from 18" to 28" in combination with four degrees of inflation pressure and loading. The mathematical model will calculate the contact area of elastic tyre on a soft surface according to Equations (6–8):

$$S = 0.77 \times b_c \times l_c \quad (6)$$

where:

l_c – contact length (Figure 3).

The contact width b_c in Equation (6) is counted according to Equation (7):

$$b_c = b_P + c_1 \times \frac{F_K}{F_{KN}} \quad (7)$$

where:

F_K – static wheel loading (N);

F_{KN} – nominal wheel loading (N);

c_1 – constant, $c_1 = 0.3–0.5$.

The contact length l_c in Equation (6) is counted according to Equation (8):

$$l_c = l_1 + l_2 = \sqrt{d \times (z + \delta) - (z + \delta)^2} + \sqrt{d \times \delta - \delta^2} \quad (8)$$

where:

z – wheel sinking [rut depth (m)];

δ – tyre deformation (m).

Model according to Febo (1987). This model represents an empirical relation for calculating the tyre contact area on a soft surface, developed on the basis of measuring characteristics of tyres used in agriculture, see Equations (9–11):

$$S = \frac{\pi}{4} \times l_c \times b_c \quad (9)$$

$$l_c = 2 \times \sqrt{d} \times \delta^j \quad (10)$$

$$b_c = b_W \times \left(1 - \exp^{-k \times \delta}\right) \quad (11)$$

where:

j – constant, for standard tractor tyres $j = 0.41$, for tyres of forest machines $j = 0.44$;

k – constant, for standard tyres $k = 33$, the value recommended for tyres of forest machines is $k = 36$ (Saarilahti 2002);

b_W – tyre pattern width (m).

Model according to Lyasko (1994). This model uses the results of research studies conducted in the Soviet Union for the calculation of the contact area of elastic tyre on the solid ground. The relation for calculating the contact area S is the same as defined above in Equation (9); contact length and width are to be calculated according to the following Equations (12–14):

$$l_c = c_3 \times \sqrt{d \times \delta - \delta^2} \quad (12)$$

$$c_3 = \frac{23}{ABS\left(\frac{d}{b_P} - 3.5\right) + 11.9} \quad (13)$$

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$$b_c = 2 \times \sqrt{\frac{b_b + h}{2.5} \times \delta - \delta^2} \quad (14)$$

where:
ABS – absolute value;
h – tyre profile height (m).

***Model according to Saarilahti* (*2002*).** This model presents the calculation of the contact area of elastic tyre on the solid ground. The calculation of the contact length l_c is based on Equation (8), when soil deformation is $z = 0$, as in Equation (15):

$$l_c = 2 \times \sqrt{d \times \delta - \delta^2} \quad (15)$$

Contact width b_c is calculated using tyre deformation δ and transversal radius r_b [Equation (16)]:

$$b_c = 2 \times \sqrt{2 \times r_b \times \delta - \delta^2} \quad (16)$$

where:
r_b – transversal radius.

The transversal radius is recommended for diagonal tyres of machines with a small side deformation, see Equation (17):

$$r_b = \frac{b_P}{2} \quad (17)$$

Field measurements

The tractor-and-trailer unit was tested at the Training Forest Enterprise Masaryk Forest Křtiny, Mendel University in Brno, which consisted of Valtra 134 tractor and Agama LV10 trailer with the effective carrying capacity of 10 t (Table 1). The front axle of the Valtra 134 tractor was equipped with Nokian 380/85-28 tyres, and the rear axle was equipped with Nokian 460/85-38 tyres. The tandem axle of the Agama LV10 trailer was equipped with Alliance 500/45-22.5 tyres. During the tests, the tyres were inflated to three pressure levels – 150 kPa, 200 kPa, and 250 kPa. The tests were performed in two variants of timber unit loading: empty trailer and trailer with a load of 4 200 kg (spruce roundwood of 4 m in length). The longitudinal slope gradient and cross slope of the solid surface was 0°.

In the tests, the radius of unloaded tyres and radius of loaded tyres were measured by an electronic calliper on the solid ground for each variant of loading and inflation pressure, and tyre footprints were taken on a large format sheet of paper. Subsequently, tyre deformations were calculated for the realised variants of measurements. Images of tyre contact areas were evaluated using the method of image analysis in the graphical programme Draft Sight (Version Standard, 2014). Each contact area was circumscribed by a curve and its surface area was calculated using the programme. At the same time, contact length l_c and contact width b_c of the footprint were also measured in the programme. Exponents of the super ellipse n (Keller 2005) were calculated according to Equations (3–5), and the curves of the super ellipse were constructed and added into the footprints. Examples of tyre footprints with the marked tyre contact area and illustrated super ellipse are presented in Figure 4. The measured data were compared with the values calculated according to relations presented in the theoretical part.

Table 1. Types and dimensions of tyres, axle load

Parameters – axle load	Tractor Valtra 134		Trailer Agama LV 10	
	front axle	rear axle	front axle	rear axle
Empty trailer (kg)	15 845	19 375	9 418	10 010
Loaded trailer (kg)	13 685	21 582	20 012	19 965
Parameters – tyres type	Nokian 380/85-28	Nokian 460/85-38	Alliance 500/45-22.5	Alliance 500/45-22.5
Diameter (mm)	1 349	1 736	1 015	1 015
Width (mm)	370	460	500	500
Pattern width (mm)	360	450	490	490
Profile height (mm)	323	391	225	225

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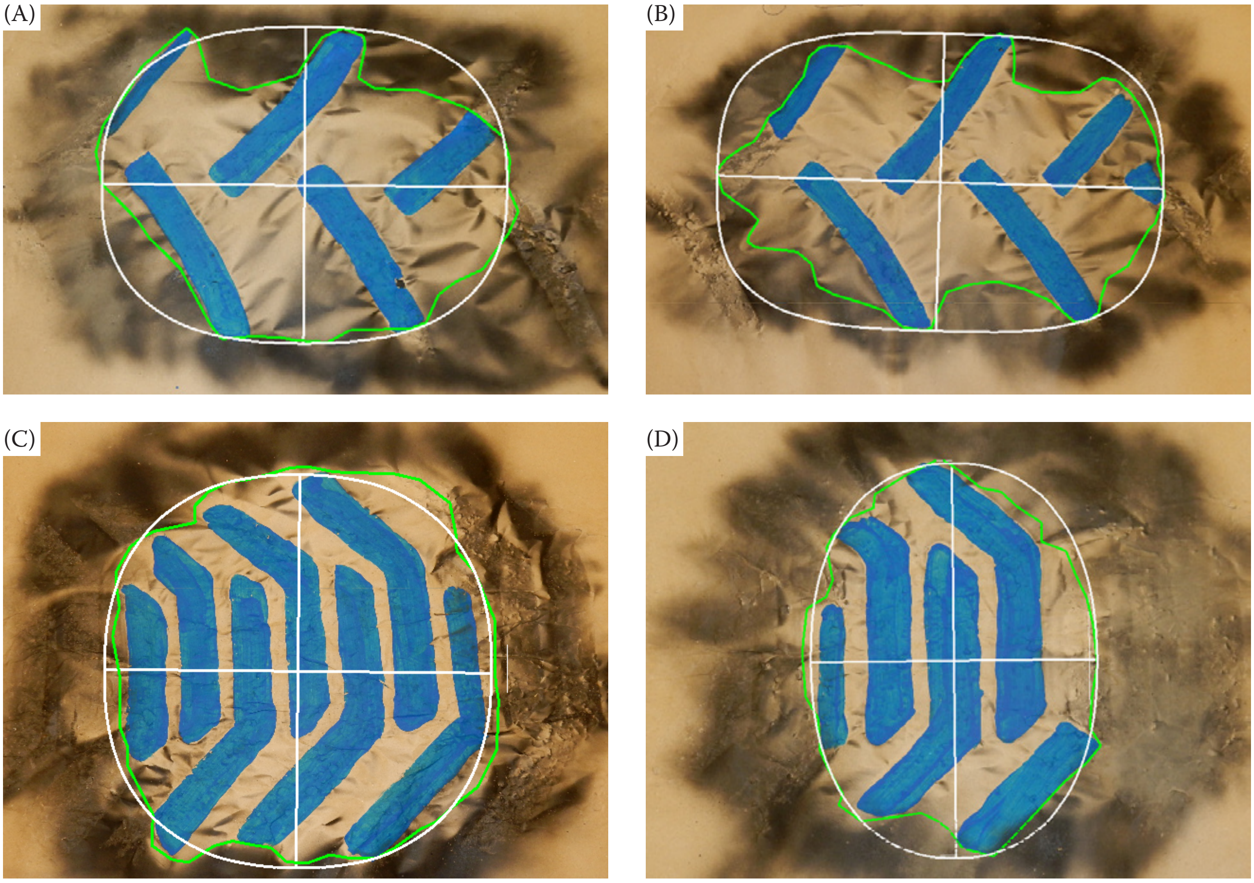

Figure 4. Images of tyre footprints of tractor and trailer in variants 'empty' and 'loaded' with the circumscribed contact area and indicated super ellipse: (A) tractor (front tyre), inflation pressure 150 kPa, loaded trailer; (B) tractor (rear tyre), inflation pressure 150 kPa, loaded trailer; (C) trailer (front tyre), inflation pressure 200 kPa, loaded; and (D) trailer (front tyre), inflation pressure 200 kPa, empty

RESULTS

Information about the tyre deformation (due to its loading) and tyre contact area for three different variants of tyre inflation and two variants of axle loading is given in Tables 2 and 3.

The measured and calculated values of tyre footprint length, width and contact area for the tractor-and-trailer unit with the empty and loaded trailer are shown in Figures 5–10. The match of measured and calculated data is presented in Tables 4–6. Figures 5 and 6 present results of calculations of tyre footprint length according to Schwanghart (1991) and Saarilahti (2002) shown together as one value as the method of calculation is identical: Equation (8) – Schwanghart (1991) is identical to Equation (15) – Saarilahti (2002) when the ground deformation is zero ($z = 0$).

Figures 5 and 6 demonstrate a relatively good match of results of the Schwanghart (1991) – 86% and Lyasko (1994) – 89% calculation models with the measured values of tyres of an empty and loaded trailer, which were wide tyres with the profile of tyre width and profile height being $b/h = 2.13$. The calculation model of Febo (1987) has a lower match (59%) in this case, which is given by the constant j in the relation, being empirically determined for individual types of tyres, while a constant recommended for the tyres of forest machines is $j = 0.44$.

In tractor tyres, calculation models by Schwanghart (1991) and Lyasko (1994) show a lower match with the measured data (62% and 59%, respectively). The standard tractor tyres had a ratio of width to profile height $b/h = 1.16$ and 1.15. The Febo (1987) calculation model exhibits a better match (75%), which is given by a better adjustment of the constant j (in standard tractor tyres $j = 0.41$).

For the resulting calculation of the super ellipse, we selected Equation (8) – Schwanghart (1991) in which the match with the measured values for

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Table 2. Deformation of tyres and parameters of footprints of tyres in unloaded trailer

Tyre parameters	Tractor						Trailer					
	front axle			rear axle			front axle			rear axle		
Tyre inflation pressure (kPa)	250	200	150	250	200	150	250	200	150	250	200	150
Tyre deformation (mm)	7	25	45	7	12	15	22	29	36	22	29	36
Footprint length (mm)	434	450	480	522	539	574	241	254	273	258	270	313
Footprint width (mm)	355	369	380	424	428	429	312	350	363	335	348	351
Contact area (m^2)	0.089	0.105	0.115	0.147	0.154	0.194	0.065	0.068	0.079	0.077	0.088	0.091

Table 3. Deformation of tyres and parameters of footprints of tyres in loaded trailer

Tyre parameters	Tractor						Trailer					
	front axle			rear axle			front axle			rear axle		
Tyre inflation pressure (kPa)	250	200	150	250	200	150	250	200	150	250	200	150
Tyre deformation (mm)	3	7	35	9	30	40	38	45	57	38	45	57
Footprint length (mm)	417	445	475	543	555	653	396	403	485	374	399	452
Footprint width (mm)	346	363	371	422	436	443	392	409	436	368	411	418
Contact area (m^2)	0.097	0.100	0.124	0.151	0.171	0.194	0.134	0.137	0.189	0.112	0.139	0.152

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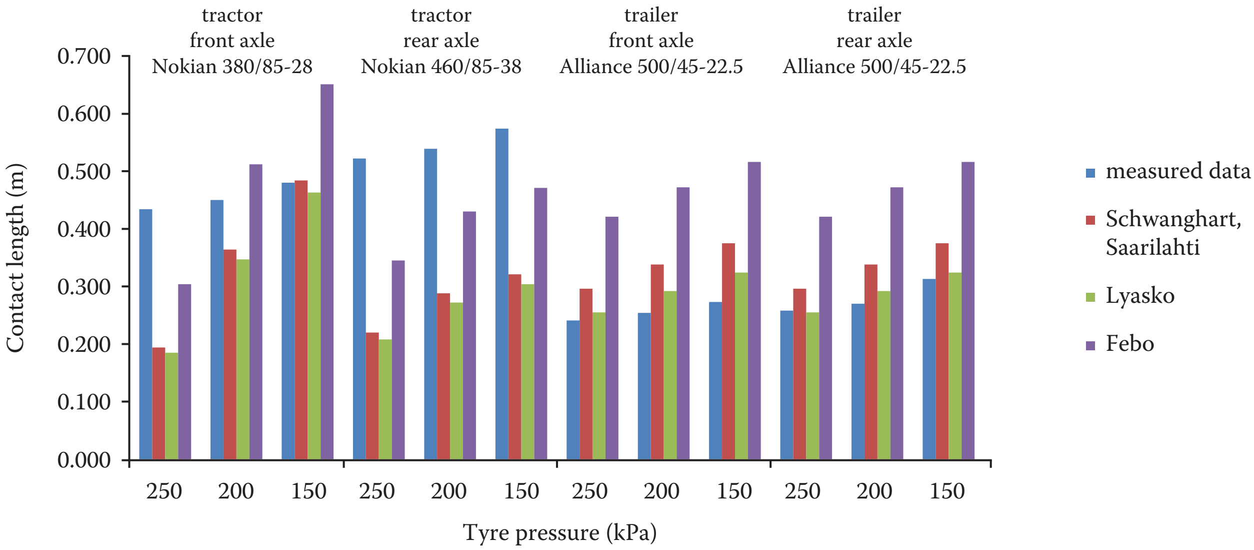

Figure 5. Comparison of calculated tyre footprint lengths with measured values – Empty trailer

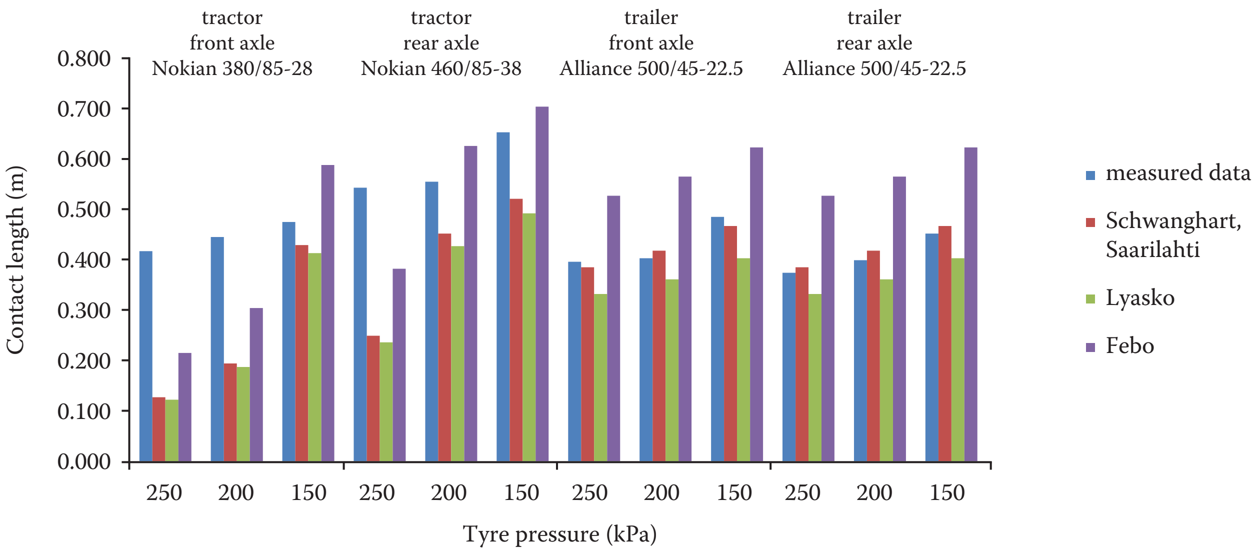

Figure 6. Comparison of calculated tyre footprint lengths with measured values – Loaded trailer

tyres of tractor and trailer was 75%. The relation is also entered by wheel sinking (rut depth) as a parameter, and it is therefore applicable also in conditions of the ground with lower bearing capacity.

According to relations published by Lyasko – Equation (14), Febo – Equation (11), and Saarilahti – Equation (16), the measured values of tyre footprint width (Figures 3 and 4) markedly differ from the calculated values. Results of measurements indicate that at the inflation pressure of 250 kPa, the tyre footprint width is smaller than the total tyre width and increases slightly with the decreasing inflation pressure. Results according to Equation (7) – Schwanghart (1991) were closest to the measured values (match 74%). In this case, however, the equation is entered only by the tyre width b, wheel load F_K, and nominal wheel load F_{KN}, with the tyre footprint width b_c being identical at different inflation pressure. In the other authors (Febo 1987; Lyasko 1994; Saarilahti 2002), the tyre deformation δ also enters the relations as a variable, which indirectly expresses the effect of different inflation pressures; in all cases, however, the calculated tyre footprint width b_c was significantly lower than the measured values.

The difference between the calculated and measured values can be explained by dimensions and type of tyres, based on which the constants in calculation models were set. For example, in Equa-

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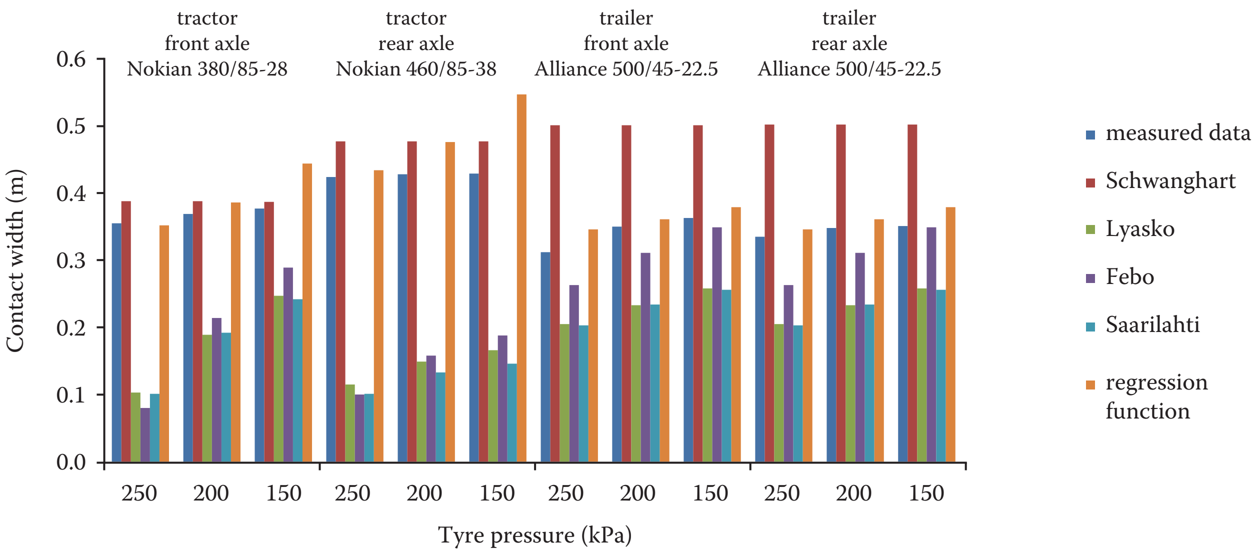

Figure 7. Comparison of calculated tyre footprint width with the measured values – Empty trailer

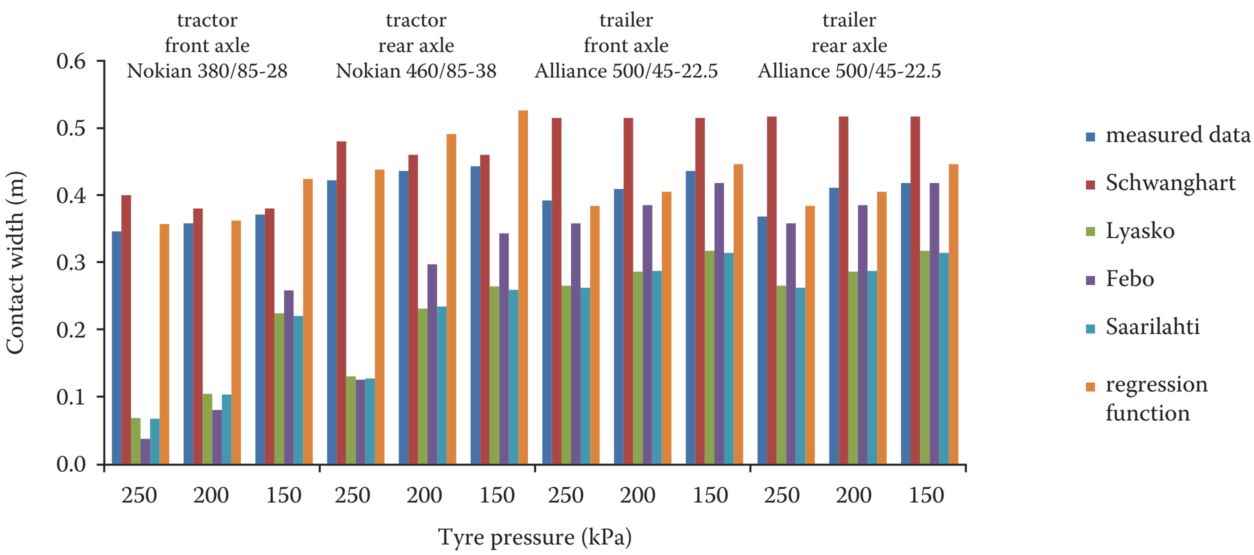

Figure 8. Comparison of calculated tyre footprint width with the measured values – Loaded trailer

tion (11) – Febo (1987), it is the constant $k = 36$, expressing the type of tyre, and in Equation (16) – Saarilahti (2002), it is the transverse radius r_b, for the value of which it is recommended to use the relation $r_b = b_P/2$.

In order to achieve a better match of the measured and calculated values of tyre footprint width, a regression function was created based on the measured data, into which a ratio of tyre width to profile height b_P/h and deformation δ enter as variables, as in Equation (18):

$$b_c = \left(-0.28 \times \frac{b_P}{h} + 1.25\right) \times \left(77 \times \delta^2 + 3 \times \delta + 1\right) \quad (18)$$

Notwithstanding the different dimensions of tyres, results calculated according to the equation exhibit a match with the measured data of 94% ($r^2 = 0.953$). The results are shown in Figures 7 and 8 under 'regression function'.

In addition to the results of tyre contact area calculations from different authors, Figures 9 and 10 also present two results of the calculation using the super ellipse. The first calculation (Super ellipse 1) makes use of tyre footprint length and width obtained from the measured data. The second calculation (Super ellipse 2) makes use of tyre footprint length and width calculated according to Equation (8) – Schwanghart (2002) and Equation (18) – regression function.

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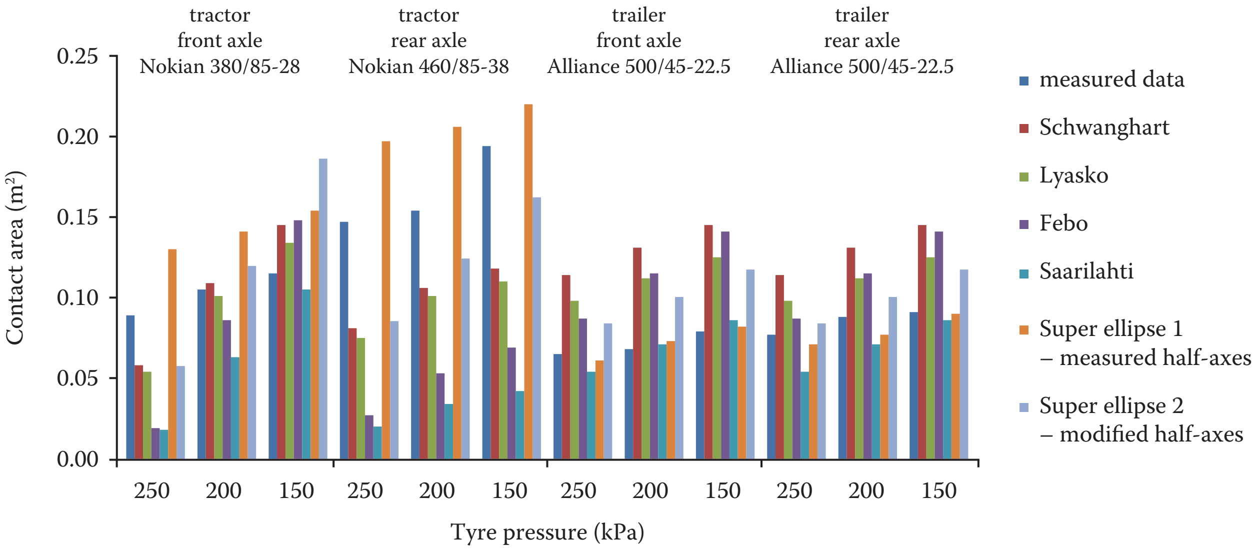

Figure 9. Comparison of measured tyre contact area with the measured values – Empty trailer

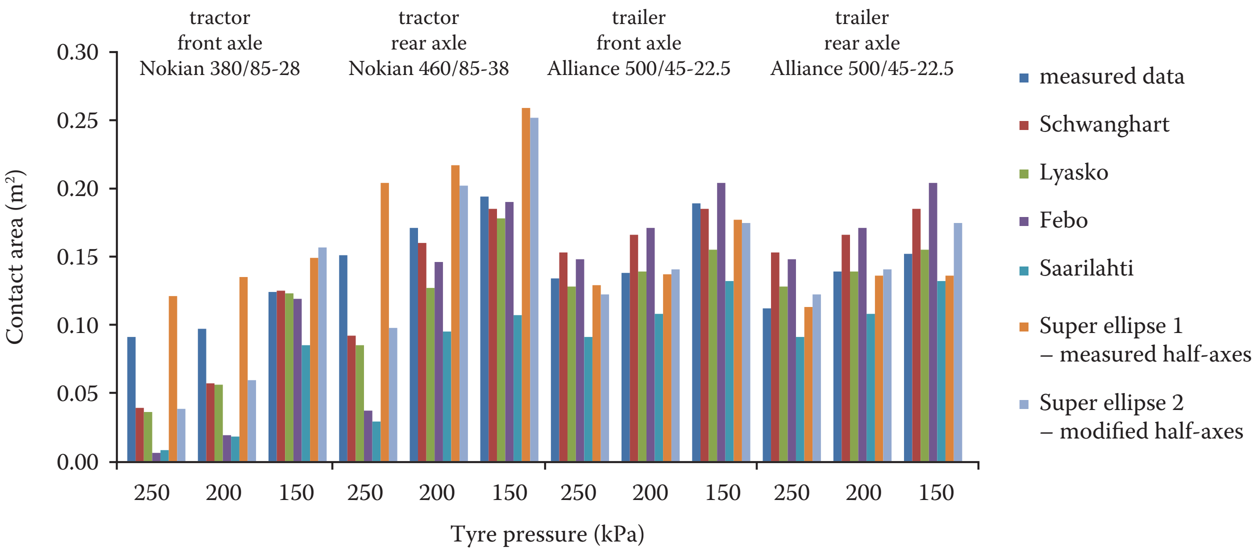

Figure 10. Comparison of calculated tyre contact area with the measured values – Loaded trailer

Figures 9 and 10 show an increasing tyre contact area with increasing tyre deformation caused by a greater load or by reduced inflation pressure. This information is in line with the results of research published in references (e.g. Komandi 1976; Schjøning et al. 2008).

Similarly like in the tyre footprint length, the tyre contact area exhibits a better match of measured and calculated values in the trailer tyres (Super ellipse 1 – 95%, Super ellipse 2 – 82%). These tyres feature a finer pattern with less protruding blocks and a larger effective area (Figures 4C and 4D). The shape of the super ellipse indicated in the figures, calculated using the exponent *n* according to Equation (3) – Keller (2005), represents an ideal contact area which is somewhat larger than the real contact area circumscribed by the green curve in the figure. Comparisons of measured and calculated values in Figures 9 and 10 also show a certain overrating of the calculated contact area, which is lower in the tyre under load. The super ellipse curve, calculated using the exponent *n* according to Equation (3) – Keller (2005), accurately copies the outer contact area shape, namely in the loaded tyre (Figure 4C). In spite of the fact that Equation (3) – Keller (2005) for the calculation of this exponent is entered only by tyre dimensions and is not dependent on tyre deformation, the shape

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Table 4. Match of measured and calculated values for the parameter of tyre footprint length

Match of data (%)	Measured values	Schwanghart, Saarilahti Equation (8)	Lyasko Equation (13)	Febo Equation (10)
Footprint length of tractor and trailer tyres	100	75	74	67
Footprint length of tractor tyres	100	62	59	75
Footprint length of trailer tyres	100	86	89	59

Table 5. Match of measured and calculated values for the parameter of tyre footprint width

Match of data (%)	Measured values	Schwanghart Equation (7)	Lyasko Equation (14)	Febo Equation (11)	Saarilahti Equation (16)	Regression function Equation (18)
Footprint width	100	77	50	69	58	94

Table 6. Match of measured and calculated values for the parameter of tyre contact area

Match of data (%)	Measured values	Schwanghart Equation (6)	Lyasko Equation (12)	Febo Equation (9)	Saarilahti Equation (6)	Super ellipse 1 Equation (2)	Super ellipse 2 Equation (2)
Contact area (tractor + trailer)	100	65	72	63	60	81	75
Contact area (trailer)	100	57	75	66	81	95	82
Contact area (tractor)	100	73	69	60	38	68	67

of the calculated super ellipse corresponds well with the contours of the contact area of both loaded and unloaded tyres.

In tractor tyres, the best match with the measured values was shown by Equation (6) – Schwanghart (1991; 73%). Calculations using the super ellipse exhibited a lower match (Super ellipse 1 – 68%, Super ellipse 2 – 67%).

The markedly protruding pattern blocks of these tyres create an effective surface area when in contact with solid ground, which is significantly smaller than the real contact area circumscribed by a green curve in Figures 4A and 4B. Also, the difference between the ideal contact area, which is represented by super ellipse, and the measured values of the real contact area is greater than in the trailer tyres with a finer pattern. However, driving across the soil with lower bearing capacity will result in tyre sinking, the contact area will be larger than that shown in Figures 4A and 4B, and the match of measured and calculated values is likely to be more favourable. In conditions with the deployment of machines, the calculation of the tyre contact area is more meaningful in soils with lower bearing capacity where it is one of the main parameters for determining the contact pressure with the soil and hence the degree of undesirable soil compaction.

The overall match of measured and calculated data for tractor and trailer tyres is the highest in Super ellipse 1 with the measured half-axes (81%). In Super ellipse 2 with the calculated half-axes, the value is lower (75%), as the calculated input parameters (contact length and contact width) have a certain deviation from the measured values, which shows in the resulting calculation. In spite of that, however, this method of calculation represents a better match than the other calculation models (Febo 1987; Schwanghart 1991; Lyasko 1994; Saarilahti 2002).

In the Discussion chapter, some empirical models for calculating the contact area are compared with the measured data, which are not entered by contact length and contact width as variables. Models selected for the comparison were those developed by Komandi (1976) – Equation (19), Diserens (2009, 2011) – Equations (21–23), and Grečenko (1995) – Equation (20). The calculation models are presented in Table 7, results of the comparison in diagrams are shown in Figures 11–13, and the match with the measured data is shown in Table 8.

https://doi.org/10.17221/109/2023-JFS

Table 7. Calculation models

Source	Calculation model	
Komandi (1976)	$$S = \frac{\left[c_4 \times F_K^{\ 0.7} \times \left(\frac{b_P}{d}\right)^{0.5}\right]}{p_i^{\ 0.45}}$$	(19)
Grečenko (1995)	$$S = c_5 \times \left(\frac{F_K}{F_{KN}}\right)^{\frac{2}{3}} \times b_P \times \left[r_s \times \left(d - 2 \times r_s\right)^2\right]^{\frac{1}{3}}$$	(20)
	$$S = 0.191 \times TS + 4.6 \times 10^{-3} \times F_K - 14.8 \times 10^{-5} \times p_i$$	(21)
Diserens (2009, 2011)	$$S = 0.130 \times TS + 9.2 \times 10^{-3} \times F_K - 53.5 \times 10^{-5} \times p_i$$	(22)
	$$S = 0.1174 \times TS + 6.6 \times 10^{-3} \times F_K - 18.3 \times 10^{-5} \times p_i$$	(23)

b_P – tyre width (m); c_4 – coefficient expressing the soil type (for good ground bearing capacity c_4 = 0.3); c_5 – coefficient expressing the tyre type (for tyres of agricultural machines c_5 = 1.65); d – tyre diameter (m); F_K – static wheel loading (N); F_{KN} – nominal wheel loading (N); p_i – inflation pressure (kPa); r_s – radius of loaded tyre (m); S – super ellipse area (m^2); TS – coefficient expressing the tyre size $TS = b_P \times d$ (m^2)

Table 8. Match of measured and calculated values for the parameter of tyre contact area

Match of data (%)	Measured values	Komandi Equation (19)	Grečenko Equation (20)	Diserens Equations (21–23)
Contact area (tractor + trailer)	100	81	53	82
Contact area (trailer tyres)	100	82	60	87
Contact area (tractor tyres)	100	80	39	80

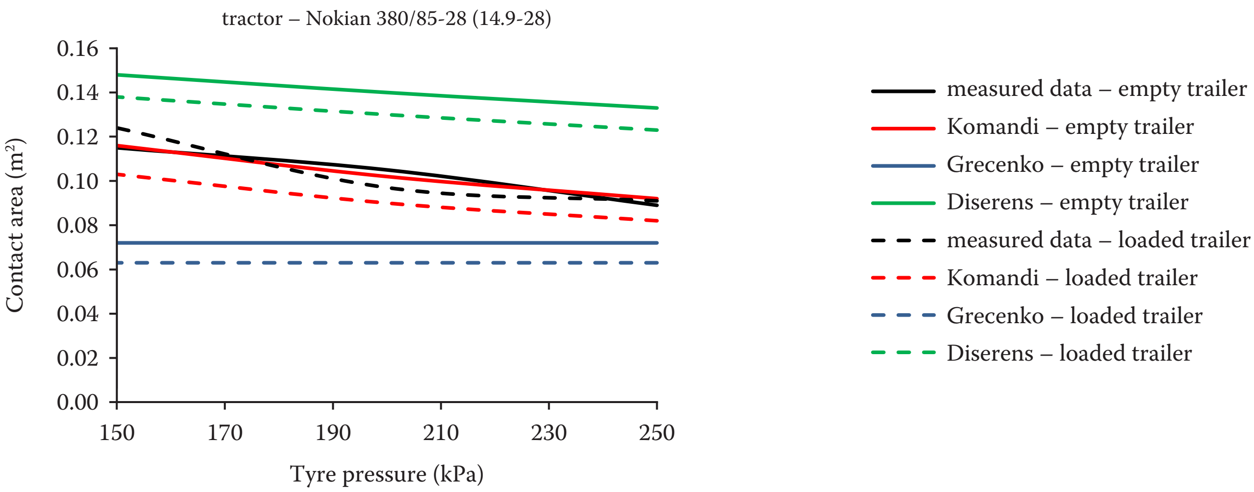

Figure 11. Comparison of calculated contact area with the measured values – Tractor front axle

https://doi.org/10.17221/109/2023-JFS

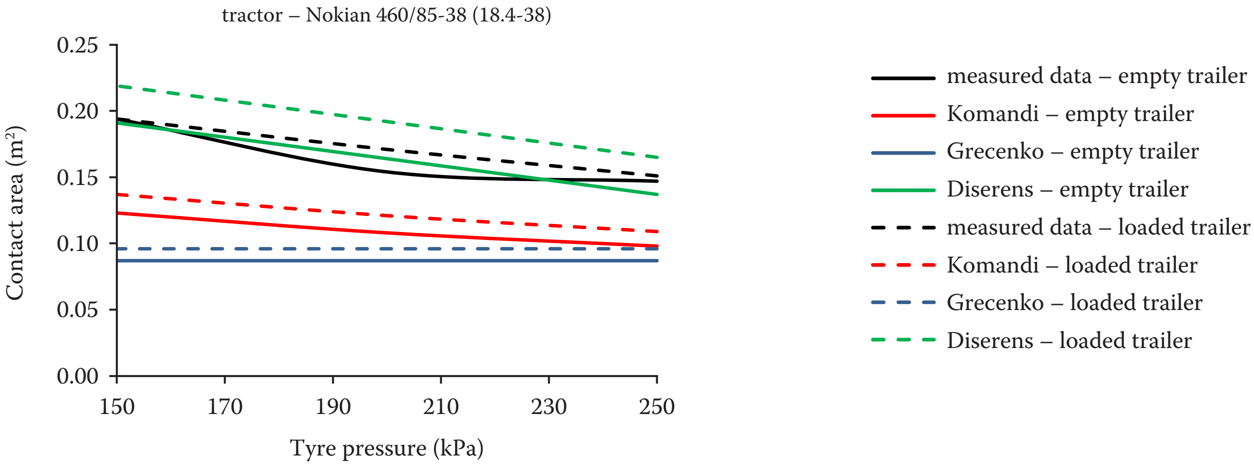

Figure 12. Comparison of calculated contact areas with the measured values – Tractor rear axle

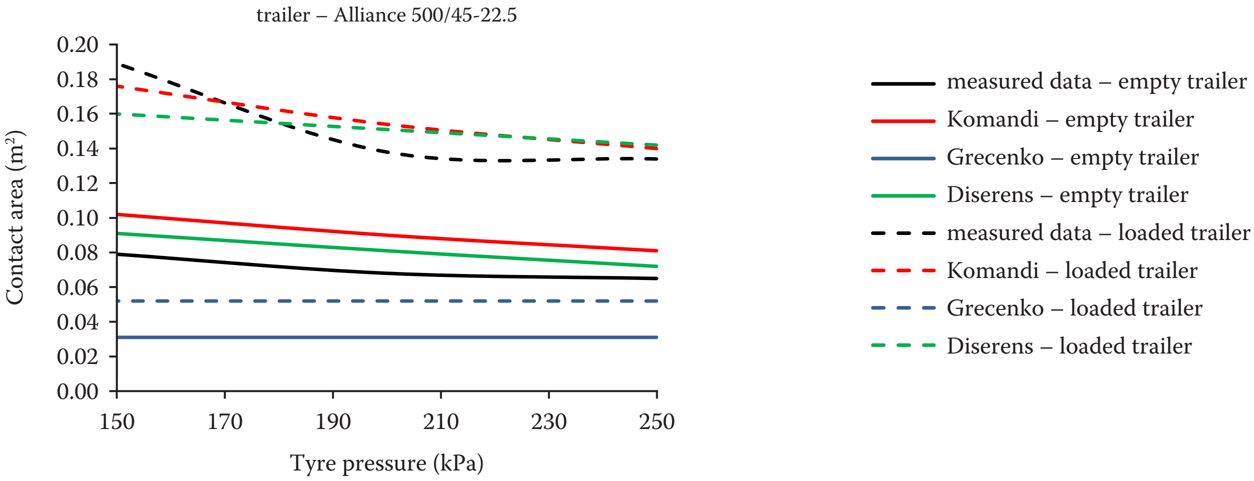

Figure 13. Comparison of calculated contact areas with the measured values – Trailer tyres

DISCUSSION

Komandi (1976) measured the contact area of diagonal tyres on several soil types. The relation for calculating the contact area is entered by the coefficient c_4, expressing the type of soil; the value chosen in the calculation for bearing soil was $c_4 = 0.3$. Although the relation was worked out based on the measurements on several soil types, it expresses a very good match (81%) with the values measured on the hard ground and coefficient $c_4 = 0.3$.

Grečenko (1995) developed a number of relations based on the measurement of contact area and data from the catalogue sheets of tyres, which are entered in addition to the tyre size also by a correction coefficient expressing the ratio of real wheel load to nominal load presented in the tyre catalogue.

Compared with the measured values, contact areas calculated according to Equation (20) are too low (53%) and remain the same under changing inflation pressure. The difference between the calculated and measured data can be caused by the fact that this calculation model specifies values from the catalogue of tyres (namely on static radius r_s), which relate to new unworn tyres.

Diserens (2009, 2011) measured contact areas in 24 types of tyres representing sizes from 16'' to 38''. He classified the tyres according to the parameter 'tyre size' (*TS*) into three groups – $TS < 0.6$; $TS = 0.6–1.2$; $TS > 1.2$, developing a regression function to calculate the contact area for each of them. In addition to the *TS* parameter, the relations are entered also by the parameters of wheel load F_K and inflation pressure p_i. The cal-

https://doi.org/10.17221/109/2023-JFS

culation made use of relations from the group $TS < 0.6$ – front tractor tyres described by Equation (21), $TS > 1.2$ – rear tractor tyres described by Equation (22) – Diserens et al. (2011) and the relation for diagonal tyres was used for trailer tyres according to Equation (23) – Diserens (2009).

Values calculated according to Equations (21–23) exhibit a very good match with the measured data (82%) although they were obtained neither on the basis of the geometry of tyre contact with the soil nor on the basis of the analysis of tyre footprint shape.

Rosca (2014) used the calculation of the tyre contact area by means of a super ellipse. In his work, he considered the super ellipse exponent $n = 3.5$ to be adequate for calculating the contact area in a wide range of tyres. The value approximately agrees with the value of the super ellipse exponent $n = 3.2$, calculated for the rear tractor tyre 460/85-38 of the tested tractor-and-trailer unit.

CONCLUSION

Calculation models help determine the size of the tyre contact area with the soil surface even before the machine is put into operation. Determining the contact area of the tyre is important in view of the efficiency of engine power transfer to the soil surface, the formation of possible erosion grooves and the risk of the soil profile compaction. In this paper, we compared the measured values of the contact area in tractor and trailer tyres with the calculation models developed by several authors. The comparison of computational models is important for choosing the model whose results achieve the best agreement with the measured data. In the first step, the comparison included calculation models using the tyre contact length and tyre contact width as input parameters. At the same time, the comparison also included the calculation of the contact area with the use of a super ellipse in which the exponent n was calculated, determining its shape for each tyre type.

Based on the measured values of the tyre contact width, a relation was developed for its calculation according to tyre size and tyre deformation. The match of the relation with the measured values is $r^2 = 0.994$.

The calculation of the tyre contact area by means of a super ellipse for three different tyre types exhibits match with the measured values 81% in the case of a super ellipse with the measured half-axes and 75% in the case of a super ellipse with the calculated half-axes.

In the second step, other empirical calculation models were compared with the measured values, which came into existence based on the statistical elaboration of results from the measurement of contact area in several tyres. Some of these models show a very good match with the measured data (up to 82%), which is comparable to or even higher than the calculation by means of the super ellipse. In our opinion, however, when the tyre deformation is entered, the calculation model using the super ellipse is more appropriate for determining the contact area size as it is based on the geometry of the tyre contact with the ground.

As the super ellipse exponent n, calculated according to Equation (3) – Keller (2005) determining its shape, depends only on the tyre dimensions (width b_P and diameter d), it would be useful to complement the relation also with the dependence on tyre deformation δ in the future research. The modified relation would better express the outer tyre footprint shape and the opinions of some other authors would be verified who claim that the shape of the tyre footprint ranges from ellipse to rectangle in dependence on its loading (deformation) and inflation pressure.

REFERENCES

Abeels P.F.J. (1994): Mechanisation of the forest operations and impacts on the environment. In: Interactive workshop and seminar FORSITRISK, soil, tree, machine interactions, Feldafing, July 4–8, 1994.

Anifantis A.S., Cutini M., Bietresato M. (2020): An experimental-numerical approach for modelling the mechanical behaviour of a pneumatic tyre for agricultural machines. Applied Sciences, 10: 3481.

Błaszkiewicz Z. (1990): A method for the determination of the contact area between a tyre and the ground. Journal of Terramechanics, 27: 263–282.

Derafshpour S., Valizadeh M., Mardani A., Saray M.T. (2019): A novel system developed based on image processing techniques for dynamical measurement of tyre-surface contact area. Measurement, 139: 270–276.

Diserens E. (2009): Calculating the contact area of trailer tyres in the field. Soil and Tillage Research, 103: 302–309.

Diserens E., Défossez P., Duboisset A., Alaoui A. (2011): Prediction of the contact area of agricultural traction tyres on firm soil. Biosystems Engineering, 110: 73–82.

https://doi.org/10.17221/109/2023-JFS

Farhadi P., Golmohammadi A., Sharifi A., Shahgholi G. (2018): Potential of three-dimensional footprint mold in investigating the effect of tractor tyre contact volume changes on rolling resistance. Journal of Terramechanics, 78: 63–72.

Febo P. (1987): Contact area tests of a new wide section agricultural tyre. Proceedings of the 9th International ISTVS Conference. Volume 1, Barcelona, Aug 31–Sept 4, 1987: 236–243.

Grečenko A. (1995): Tyre footprint area on hard ground computed from catalogue values. Journal of Terramechanics, 32: 325–333.

Hallonborg U. (1996): Super ellipse as tyre-ground contact area. Journal of Terramechanics, 33: 125–132.

Keller T. (2005): A model for prediction of the contact area and the distribution of vertical stress below agricultural tyres from readily available tyre parameters. Biosystems Engineering, 92: 85–96.

Komandi G. (1976): The determination of the deflection, contact area, dimensions, and load capacity for driven pneumatic operating on concrete pavement. Journal of Terramechanics, 13: 15–20.

Komandi G. (1990): Establishment of soil-mechanical parameters which determine traction on deforming soil. Journal of Terramechanics, 72: 115–124.

Koolen A.J., Lerink P., Kurstjens D.A.G., van den Akker J.J.H., Arts W.B.M. (1992): Prediction of aspects of soil-wheel systems. Soil and Tillage Research, 24: 381–396.

Lyasko M.I. (1994): The determination of deflection and contact characteristics of a pneumatic tyre on a rigid surface. Journal of Terramechanics, 31: 239–242.

Melzi S., Negrini S., Sabbioni E. (2014): Numerical analysis of the effect of tyre characteristics, soil response and suspensions tuning on the comfort of an agricultural vehicle. Journal of Terramechanics, 55: 17–27.

Pacas B. (1983): Teorie stavebních strojů. Brno, Brno University of Technology: 244. (in Czech)

Ptak W., Czarnecki J., Brennensthul M., Lejman K., Małecka A. (2022): Evaluation of agriculture tyres deformation using innovative 3D scanning method. Agriculture, 12: 1108.

Roşca R., Cârlescu P., Ţenu I. (2014): A semi-empirical traction prediction model for an agricultural tyre, based on the super ellipse shape of the contact surface. Soil and Tillage Research, 141: 10–18.

Saarilahti M. (2002): Soil Interaction Model. Helsinki, University of Helsinki, Department of Forest Resource Management: 86.

Schjønning P., Lamandé M., Tøgersen F. A., Arvidsson J., Keller T. (2008): Modelling effects of tyre inflation pressure on the stress distribution near the tyre-soil interface. Biosystems Engineering, 99: 119–133.

Schwanghart H. (1991): Measurement of contact area, contact pressure and compaction under tyres in soft soil. Proceedings of the 10th ISTVS Conference. Volume 1. Kobe, Aug 20–24, 1990: 193–204.

Šmerda T., Bauer F. (2010): Vliv otisku pneumatiky na energeticko-výkonnostní parametry traktorové soupravy. Research project NAZV MZe ČR QF 4080. Brno, Mendel University in Brno: 14. (in Czech)

Söhne W. (1958): Fundamentals of pressure distribution and soil compaction under tractor tires. Agricultural Engineering, 39: 276–281.

Spíchal L. (2020): Superelipsa a superformule. Mathematics-Physics-Informatics, 29: 54–68. (in Czech)

Upadhyaya S.K., Wulfsohn D. (1990): Relationship between tyre deflection characteristics and 2-D tyre contact area. Transactions of the ASAE, 33: 25–30.

Wulfsohn D., Upadhyaya S.K. (1992): Determination of dynamic three-dimensional soil-tyre contact profile. Journal of Terramechanics, 29: 433–464.

Received: October 2, 2023
Accepted: January 3, 2024